

# Comparison of Forecasting Methods for Energy Demands in Single Family Homes

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## Abstract

The integration of renewable energy sources in single family homes is challenging. Advance knowledge of the demand of electrical energy, heat, and domestic hot water (DHW) is useful to schedule projectable devices like heat pumps. In this work, we consider demand time series for heat and DHW from 2018 for a single family home in Germany. We compare different forecasting methods to predict such demands for the next day. While the 1-day-back forecast method led to the prediction of heat demand, the N-day-average performed best for DHW demand when Unbiased Exponentially Moving Average (UEMA) is used with a memory of 2.5 days. This is surprising as these forecasting methods are very simple and do not leverage additional information sources such as weather forecasts.

## 1 Introduction

Energy optimization for single family homes requires predictions of future energy demands, typically for heat, domestic hot water (DHW), and electrical power. An example is the control of a heat pump which is fueled by the power grid and photovoltaic energy from the roof top. The latter should be well utilized, but switching cycles for the heat pump must be kept low to ensure a long lifetime of the heat pump. This optimization problem requires knowledge of the energy demand at least one day in advance.

While it is easy to predict energy demands for large neighborhoods consisting of hundreds of units, it is harder for single family homes. In this work, we evaluate the appropriateness of different forecasting methods for this task. We utilize them to create models from historical energy data of a single family home and predict those demands based on these models. We compare the suitability of the forecasting approaches by the error between their predictions and the historical data.

The paper is structured as follows. Section 2 gives an overview of related work. In Section 3 we describe the studied forecasting methods. Section 4 presents and investigates the data set. An evaluation of the forecasting methods is presented in Section 5. We conclude this work in Section 6.

## 2 Related Work

Aydinalp et al. developed and evaluated a neural network approach to model residential energy consumption in their paper [5]. They adopted their model to heat and DHW demand and evaluated the accuracy of the predictions. They focused on the construction of a good neural network model and on features, but did not compare different approaches.

Lomet et al. [3] investigated the DHW consumption of sin-

gle family homes. They analysed real data from such housing units and developed an ARMA model to forecast DHW demands. Their results indicate that this type of model could be suitable to forecast such demands, but they have not performed evaluations to compare different forecasting approaches for DHW demands.

Idowu et al. [4] analysed DHW and heat demand for multi-family apartments. Based on their analysis, forecasts were computed using supervised machine learning approaches. They concluded that using super-vector regression leads to least errors, but their evaluation lacks comparison with simpler approaches like 1-day-back.

Idowu et al. [6] also evaluated and compared more recently multiple advanced machine learning approaches to forecast heat and DHW demand in residential and industrial buildings. Their evaluations show that support vector machines are well suited to forecast both DHW and heat. However, also these evaluations lack inclusion of simpler approaches like 1-day-back.

## 3 Forecasting Methods

In this section, we give a short primer on the studied forecasting methods. Forecasting predicts values for a primary time series. Historical data for that time series may be used to estimate most probable future values. Formally, forecasting the  $t$ -th value  $\hat{y}_t$  of a known discrete time series  $y_0, y_1, \dots, y_{t-1}$  is the computation of some forecasting function

$$\hat{y}_t = F(y_0, y_1, \dots, y_{t-1}).$$

Sometimes forecasting is not only based on historical data of the primary time series but also on a secondary time series  $x_0, x_1, \dots, x_t$  which is correlated with the primary time series. Thereby,  $x_0, x_1, \dots, x_{t-1}$  is historical and  $x_t$  is predicted. For example, the demand for heat (primary time series) may strongly correlate with the outside temperature

(secondary time series). Then, the secondary time series helps to forecast the desired value  $\hat{y}_t$  of the primary time series:

$$\hat{y}_t = F(y_0, y_1, \dots, y_{t-1}, x_0, \dots, x_t).$$

Different forecasting methods can be used to compute  $\hat{y}_t$ . In the following, we describe several simple forecasting methods that we evaluate later in this paper.

### 3.1 N-Day-Back

The N-day-back forecast method utilizes the time series of the  $N^{\text{th}}$  preceding day (historical data) as forecast of the next day. Let  $n$  be the number of data points per day. Then the forecasting function can be described as  $F(y_0, y_1, \dots, y_{t-1}) := y_{t-(N \cdot n)}$ . A special case is the 1-day-back forecast method which uses the historical time series of the previous day as forecast of the next day.

### 3.2 N-Day-Average

For the N-day-average method, we take the average demand of  $N$  preceding days as a demand forecast of the next day. Thereby, we utilize two different averaging methods presented in [1].

#### 3.2.1 Window Moving Average (WMA)

WMA defines a window containing the last  $W$  data points and computes their arithmetic mean. Thus,  $W$  is an integral value. In this work, we apply WMA to daily demands and compute the average demand of the last  $W = N$  days. A memory  $M = W \cdot \Delta t$  can be defined where  $\Delta t$  is the inter-sample distance, i.e., a day in our context. The memory expresses the time over which a sample is remembered in the moving average.

#### 3.2.2 Unbiased Exponential Moving Average (UEMA)

UEMA can be calculated by  $A_t = \frac{S_t}{N_t}$  where the weighted sum  $S_t$  and the weighted number  $N_t$  are recursively defined as

$$S_t = \begin{cases} X_0 & t = 0 \\ a \cdot S_{t-1} + X_t & \text{otherwise} \end{cases}$$

$$N_t = \begin{cases} 1 & t = 0 \\ a \cdot N_{t-1} + 1 & \text{otherwise.} \end{cases}$$

Past values of the original time series  $X_t$  contribute to all future average values  $A_t$ . However, their impact decreases exponentially over time. The parameter  $a$  determines the memory of the moving average by  $M = \frac{\Delta t}{1-a}$ . Again, the memory cannot be smaller than the inter-sample distance  $\Delta t$ . However, any larger memory  $M$  is possible with  $a = 1 - \frac{\Delta t}{M}$ . In the context of N-day-average we set the memory to  $M = N$ . Therefore,  $N$  does not need to be an integral value if UEMA is used for averaging.

### 3.3 Linear Regression (LR)

LR [7] describes the dependency of the primary time series  $y_0, y_1, \dots, y_{t-1}$  on the secondary time series  $x_0, x_1, \dots, x_t$  with

a linear function  $f(x) := \beta_0 + \beta_1 \cdot x$ . This may be useful if both time series are highly correlated. The forecast value  $\hat{y}_t$  is then the mapping of  $x_t$  under this linear function. Mostly LR finds a best-fit line that minimizes the sum of squared distances for a set of data points  $(x_i, y_i)_{0 \leq i < n}$ . This line is efficient to compute by a compact formula. However, our comparative metric in Section 5 is the absolute average error  $\frac{1}{n} \sum_{0 \leq i < n} |f(x_i) - y_i|$  between the best-fit line  $f$  and a set of data points. Therefore, we prefer a simple numerical method based on nested intervals to derive appropriate parameters  $\beta_0$  and  $\beta_1$  that minimizes  $\frac{1}{n} \sum_{0 \leq i < n} |f(x_i) - y_i|$ .

### 3.4 Bounded LR (BLR)

As the LR-based best-fit line yields negative heat demands for high outside temperatures in Section 4.1, we propose BLR. It utilizes a best-fit line, but yields zero instead of negative values. Also for BLR we compute the parameters  $\beta_0$  and  $\beta_1$  for a best-fit line by minimizing the sum of absolute errors.

### 3.5 Daytime-Specific BLR

We will first apply LR and BLR based on entire days, i.e., we compute a best-fit line that takes the average daily temperature as input and yields the daily energy demand as output. As an alternative, we will apply LR and BLR based on the average temperature of the daily time intervals 0-6, 6-12, 12-18, and 18-24 o'clock and predict their energy demands. Finally, we sum up the predicted energy demands over a day to obtain the daily energy demand.

### 3.6 Smoothing

The dependency of the forecast time series on the prediction time series may be time-delayed. For example, the temperature in a building does not fall immediately when it becomes cool outside, in particular in case of good insulation. As a consequence, average temperatures smoothed over time may yield better forecasts for energy demands. Therefore, we propose to apply the LR and BLR methods based on smoothed historical data. We use UEMA (see Section 3.2.2) to smooth the data series, i.e., we use the average value  $A_t$  instead of  $X_t$ .

Smoothing variant	Time series	
	Primary	Secondary
no-smoothing	original	original
x-smoothing	smoothed	original
y-smoothing	original	smoothed
xy-smoothing	smoothed	smoothed

**Table 1** The smoothing variant determines whether model parameters are calculated based on original or smoothed times series.

We now explain several smoothing variants for LR/BLR forecasting. Secondary and/or primary historical time series may be smoothed for forecasting. The linear functions for the LR/BLR model are computed based on either original or smoothed time series. Table 1 shows 4 variants and

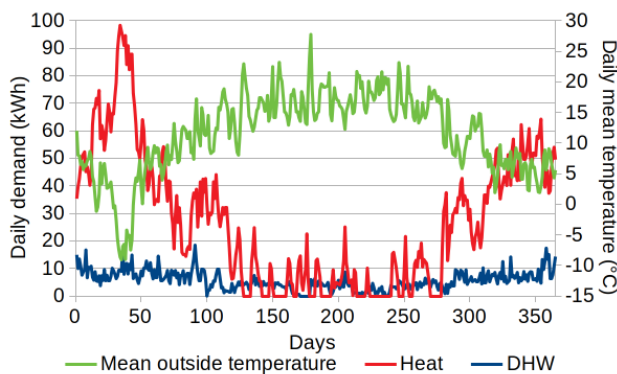
the time series based on which the parameters for the best-fit line are fitted.

For prediction purposes, the original secondary time series are utilized for no-smoothing and x-smoothing. For y-smoothing and xy-smoothing, the secondary time series  $x_0, x_1, \dots, x_t$  is smoothed including the predicted value  $x_t$  for which the corresponding value  $\hat{y}_t$  of the primary time series is to be forecast based on the computed LR/BLR model.

## 4 Data Analysis

The data set used for the evaluation in Section 5 contains real data from a single family home near Düsseldorf of the year 2018. The data set consists of time series for PV production, outside temperature, heat demand, DHW demand, and other electrical demand. The resolution is one data point per minute. In contrast, our objective in Section 5 is to forecast the heat and DHW demand for the entire next day.

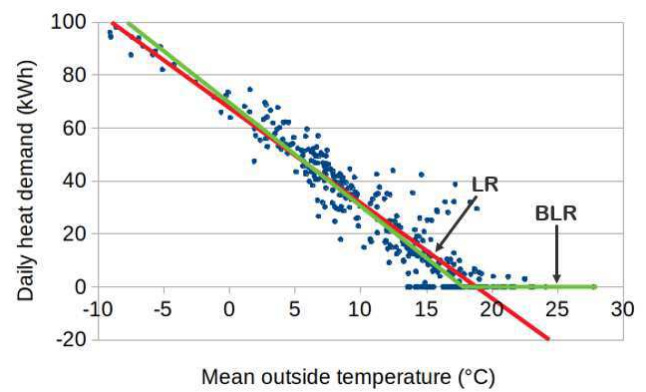
Figure 1 illustrates the data while temperature is averaged and demands are accumulated per day. The temperature is low in winter and high in summer with considerable oscillations throughout the year. In the following, we analyse the data for heat and DHW demand in more detail.



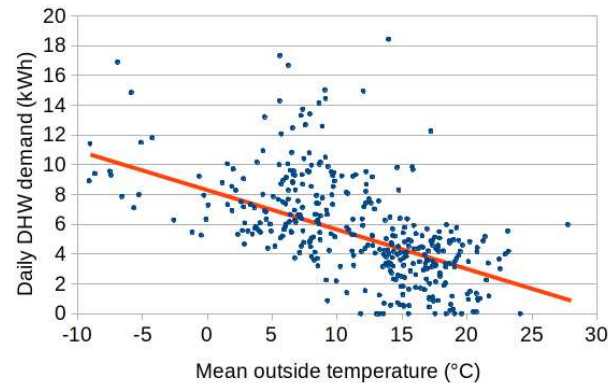
**Figure 1** Daily heat and DHW demand together with mean outside temperature per day in 2018.

### 4.1 Heat

Figure 1 shows that the energy demand for heat is a multiple of the one for DHW. In addition, we clearly see that it strongly depends on the outside temperature. We demonstrate this phenomenon by the scatter plot in Figure 2a. The coordinates of the points consist of the average temperature of a single day and the corresponding heat demand. The points are clustered along a line and we recognize the obvious trend that heat demand decreases with increasing outside temperature. This proposes a linear relation between daily heat demand and mean outside temperature. We obtain the the best-fit lines for LR with the parameters  $\beta_0 = 66.830$  and  $\beta_1 = -3.484$  and for BLR with the parameters  $\beta_0 = 69.600$  and  $\beta_1 = -3.910$ . These lines minimize the average deviation to the data points. They are also illustrated in Figure 2a. In contrast to LR, the BLR best-fit line does not yield negative heat demands.



**(a)** Heat demand with best-fit lines for LR and BLR.



**(b)** DHW demand with best-fit line for LR.

**Figure 2** Impact of mean outside temperature on the daily heat and DHW demand.

### 4.2 Domestic Hot Water (DHW)

Figure 1 shows that the demand for DHW is rather low over the year compared to the demand for heat. It oscillates over days and there is some seasonal impact.

We provide a scatter plot in Figure 2b. The points are clustered, but they are more distributed than in 2a, i.e., the linear dependency of the DHW demand on the outside temperature is weaker than the one of heat. The figure also shows the LR best-fit line which is obtained for  $\beta_0 = 9.122$  and  $\beta_1 = -0.305$ . As this line does not yield negative values in the relevant range, there is no need for BLR.

## 5 Evaluation

In this section we evaluate several forecasting methods for heat and DHW demand. First, we describe the evaluation methodology. Then, we apply the forecasting methods presented in Section 3 to the data set presented in Section 4 in order to compare their predicted heat and DHW demands.

### 5.1 Methodology

We predict the per-day heat and DHW demand for all days of 2018 in the data set. To assess the accuracy of the forecasting methods, we compute the difference of the real demands in the data set and the forecast values, and provide

absolute and relative average errors.

The N-day-back and the N-day-average methods can be applied to the data set just after aggregating the demands for entire days.

Parameter fitting is not needed for N-day-back and for N-day-average. This is different for LR and BLR. For these methods we derived the best-fit lines based on the entire data set as presented in Section 4. We use it to predict the demand for a specific day based on its average outside temperatures. That is, we predict the data based on which we calibrated the LR and BLR model. We do that due to lack of sufficient data. This is not feasible in practice and rather overrates the quality of the forecast. Nevertheless, we will see that these methods are outperformed by simpler methods.

For N-day-back, N-day-average, or smoothing variants, preceding days may be missing at the beginning of the year. Then we take the days at the end of the year as a substitute in a cyclic manner.

## 5.2 Heat

We consider forecast for heat demands. Table 2 compiles the absolute and relative average forecast errors to quantify the accuracy of various forecasting methods.

Forecast method	Abs. avg. error (kWh)	Rel. avg. error (%)
1-day-back	4.303	15.9
3-day-back	7.723	28.5
7-day-back	9.692	35.7
Linear regression	6.226	22.9
Bounded LR	5.232	19.3
Daytime-sp. BLR	5.375	19.8

**Table 2** Absolute and relative average errors for forecasts of daily heat demands; the average demand is 27.15 kWh.

The 1-day-back method leads to the least forecast errors, followed by BLR and the daytime-specific BLR method. The other methods cause significantly larger forecast errors. The 7-day-back method performs particularly badly. We tested that method as we suspected that weekly patterns in human behaviour could have a measurable impact on heat demand.

BLR clearly outperforms linear regression because it does not forecast negative values for high outside temperatures. For daytime-specific BLR we obtained daytime-specific best-fit lines with the parameters given in 3. However, daytime-specific BLR is worse than normal BLR so that its complexity does not pay off.

Interval	$\beta_0$	$\beta_1$
0-6	14.2	-1.203
6-12	25.5	-1.269
12-18	27.8	-1.536
18-0	6.2	-0.408

**Table 3** Parameters for best-fit lines for BLR-based daytime-specific forecasts.

We consider N-day-average whose forecast errors are compiled in Table 4, both for WMA and for UEMA as averaging methods. N-day-average with a memory of a single day yields 1-day-back, therefore, we see the same errors. Values for WMA can be computed only for memories that are multiples of entire days. Increasing the memory degrades predictions of heat demands both for WMA and UEMA so that N-day-average is not useful compared to 1-day-back.

Memory (d)	WMA		UEMA	
	Abs. avg. error (kWh)	Rel. avg. error (%)	Abs. avg. error (kWh)	Rel. avg. error (%)
1	4.303	15.9	4.303	15.9
1.5	-	-	4.682	17.2
2	4.990	18.4	5.053	18.6
2.5	-	-	5.362	19.8
3	5.579	20.6	5.618	20.7

**Table 4** Absolute and relative average errors for forecasts of daily heat demand using the N-day-average method with WMA and UEMA.

We investigate the potential of the smoothing variants mentioned in Table 1 to improve forecasts. For no-smoothing and y-smoothing, we take the outside temperatures of the same day in the data set as x-input for the BLR model. In case of x-smoothing and xy-smoothing, we compute a time series for smoothed temperatures in 2018 and use the smoothed temperature of the corresponding day as x-input for the BLR model.

Table 5 provides forecast results for different memories. A memory of 1 day means no smoothing. We observe that no smoothing variant improves the forecast of the simple BLR method. In contrast, increasing memory degrades forecasting results.

Memory (d)	x-smoothing (%)	y-smoothing (%)	xy-smoothing (%)
1	19.3	19.3	19.3
2	19.4	19.4	19.4
3	19.8	19.8	19.5
4	20.1	20.0	19.5

**Table 5** Relative error for the forecasts of daily heat demand; different memories are considered for x, y, and xy-smoothing in combination with BLR.

## 5.3 Domestic Hot Water (DHW)

We consider forecast of DHW demands. Table 6 compiles the absolute and relative average errors for various forecasting methods. Again, the 1-day-back method is best, followed by LR. The 3- and 7-day-back methods do not perform well. As the best-fit line for LR does not yield negative values, there is no need for BLR as LR and BLR lead to the same best-fit line under such conditions.

Forecast method	Abs. avg. error (kWh)	Rel. avg. error (%)
1-day-back	1.867	33.0
3-day-back	2.222	39.3
7-day-back	2.437	43.1
Linear regression	2.091	37.0

**Table 6** Absolute and relative average error for forecasts of daily DHW demand; the average demand is 5.653 kWh.

We have also experimented with all the presented smoothing variants but without any improvement up to large memories of 84 days.

Finally, we consider N-day-average as forecasting method. Table 7 presents the absolute and relative average errors for this method. The forecast errors depend on the specific averaging method, i.e., WMA or UEMA, and the chosen memory. UEMA yields better forecasts than WMA and the best memories are 2.5 days for UEMA and 3 days for WMA. Both methods clearly outperform even 1-day-back so that UEMA leads to the best forecasting results for DHW.

Memory (d)	WMA		UEMA	
	Abs. avg. error (kWh)	Rel. avg. error (%)	Abs. avg. error (kWh)	Rel. avg. error (%)
1	1.867	33.0	1.867	33.0
1.5	-	-	1.702	30.1
2	1.740	30.8	1.672	29.6
2.5	-	-	1.666	29.5
3	1.724	30.5	1.678	29.7
3.5	-	-	1.692	29.9
4	1.751	31.0	1.708	30.2
4.5	-	-	1.721	30.4
5	1.757	31.1	1.733	30.6
6	1.777	31.4	1.756	31.1
7	1.793	31.7	1.774	31.4
14	1.936	34.3	1.871	33.1
21	1.936	34.3	1.935	34.2
28	1.967	34.8	2.002	35.4

**Table 7** Absolute and relative average error for the forecasts of daily DHW demand using the N-day-average method with WMA and UEMA.

## 6 Conclusion

We compared different methods to predict the demand for heat and domestic hot water (DHW) for the next day in a single family home.

To predict heat demand, we obtained the best results with the very simple 1-day-back method. It clearly outperformed linear regression (LR), bounded LR (BLR), and daytime-specific BLR. This is surprising as 1-day-back does not take advantage of available weather forecast, which is in contrast to some other methods. Averaging

methods led to worse results. Smoothing-based BLR could not achieve any improvement with regard to BLR.

To forecast DHW demand, 1-day-back again outperformed LR including smoothing methods. However, the N-day-average led to even better results for small memories. We obtained the best forecasts for the UEMA averaging method with a memory of 2.5 days. This is again surprising as N-day-average does not leverage additional information like weather forecast, either.

Although we have identified best forecast methods for heat and DHW including parameters, we point out that these results have been gained from a single family home in 2018. It would be helpful to validate our findings on a larger data set, over a longer duration, and for houses with different energy demands. Moreover, it would be interesting to consider aggregated demands from multiple houses or blocks of flats. The use of machine learning approaches is certainly also worthwhile to study provided sufficient data is available.

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